OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4731:** Mechanics 4

Mark Scheme for June 2013

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Amswer given

Subject-specific Marking Instructions for GCE Mathematics (OCR) Mechanics strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awar

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark s designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full m must not be judged on the answer alone, and answers that are given in the question, especially, must be validly steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* inco Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the m award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if severa candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially under marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually suffici candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more workin… …ana… establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a … answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this w… case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the … specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is use… that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may … when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no m… sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, t… are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previous… results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permi… (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded f… intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, e… acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leade…

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A mar… 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question … not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candida… question-by-question.

f. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is nu… correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres u… particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise … specification.

When a value is given in the paper
Only accept an answer correct to at least as many significant figures as the given value. This rule should be app… case.

When a value is not given in the paper
Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct accuracy error, except for errors due to pr
which should be penalised only once in the examination.

There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accurac
question.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, th
should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what a
the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question
unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then appl
generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the c

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by cha
results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

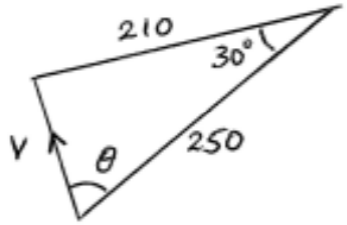
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

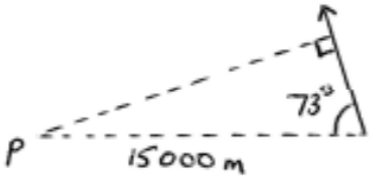
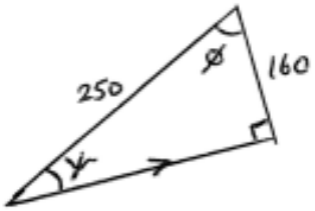
i. If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks
answers (provided, of course, that there is nothing in the wording of the question specifying that analytical metho
required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. W
with no supporting method score zero. If in doubt, consult your Team Leader.

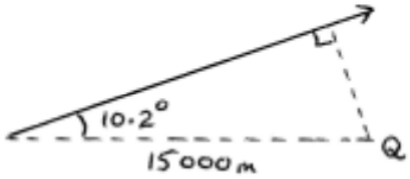
j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	Guid
1	(i)		$76 = 42 + \alpha \times 8$ Angular acceleration is $4.25\ \text{rad s}^{-2}$	M1 A1 **[2]**	Using $\omega_1 = \omega_0 + \alpha t$	
1	(ii)		$810 = 42t + 2.125t^2$ $t = \dfrac{-42 \pm \sqrt{42^2 + 4 \times 2.125 \times 810}}{2 \times 2.125}$	M1 A1	Using $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ FT Quadratic equation for t	
		OR	$\omega_1^2 = 42^2 + 2 \times 4.25 \times 810$ $\omega_1 = 93$ $93 = 42 + 4.25t$		M1 $\omega_1^2 = \omega_0^2 + 2\alpha\theta$ and $\omega_1 = \omega_0 + \alpha t$ A1 FT Equation for t	Or equivalen
			Time is 12 s	A1 **[3]**		
2			$M = \int_0^a k\left(4 - \sqrt{\frac{x}{a}}\right)\text{d}x$	M1	For $\int\left(4 - \sqrt{\frac{x}{a}}\right)\text{d}x$	
			$= k\left[\ 4x - \frac{2}{3}a^{-\frac{1}{2}}x^{\frac{3}{2}}\ \right]_0^a \quad \left(= \frac{10}{3}ka\right)$	A1	For $4x - \frac{2}{3}a^{-\frac{1}{2}}x^{\frac{3}{2}}$	
			$M\bar{x} = \int_0^a k\left(4 - \sqrt{\frac{x}{a}}\right)x\,\text{d}x$	M1	For $\int\left(4 - \sqrt{\frac{x}{a}}\right)x\,\text{d}x$	
			$= k\left[\ 2x^2 - \frac{2}{5}a^{-\frac{1}{2}}x^{\frac{5}{2}}\ \right]_0^a \quad \left(= \frac{8}{5}ka^2\right)$	A2	For $2x^2 - \frac{2}{5}a^{-\frac{1}{2}}x^{\frac{5}{2}}$	Give A1 for
			$\bar{x} = \dfrac{\frac{8}{5}ka^2}{\frac{10}{3}ka}$	M1	Dependent on previous M1M1	
			$= \frac{12}{25}a = 0.48a$	A1 **[7]**		

Question			Answer	Marks	Guid	
3			$M = \rho\int \pi y^2 \,\mathrm{d}x = \int_0^{a\ln 2} \rho\pi a^2 \mathrm{e}^{\frac{2x}{a}}\,\mathrm{d}x$	M1	For $\int\left(\mathrm{e}^{\frac{x}{a}}\right)^2 \mathrm{d}x$	
			$= \left[\rho\pi\frac{a^3}{2}\mathrm{e}^{\frac{2x}{a}}\right]_0^{a\ln 2} = \frac{3}{2}\rho\pi a^3$	A1	For equation $M = \frac{3}{2}\rho\pi a^3$ oe	
			$I = \sum\frac{1}{2}(\rho\pi y^2\delta x)y^2 = \frac{1}{2}\rho\pi\int y^4\mathrm{d}x$	M1	For $\int y^4\mathrm{d}x$	
			$= \int_0^{a\ln 2} \frac{1}{2}\rho\pi\left(a\mathrm{e}^{\frac{x}{a}}\right)^4 \mathrm{d}x$	A1	Correct integral expression for I	
			$= \left[\frac{1}{2}\rho\pi\frac{a^5}{4}\mathrm{e}^{\frac{4x}{a}}\right]_0^{a\ln 2}$	M1	Integral is a multiple of $\mathrm{e}^{\frac{4x}{a}}$	
			$= \frac{15}{8}\rho\pi a^5$	A1		
			$= \frac{15}{8}\left(\frac{2M}{3\pi a^3}\right)\pi a^5$	M1	Obtaining I in terms of M and a	Dependent c
			$= \frac{5}{4}Ma^2$	A1	A0 for $\frac{\mathrm{e}^{4\ln 2}-1}{4(\mathrm{e}^{2\ln 2}-1)}Ma^2$ etc	*Accept* $\frac{15}{12}M$
				[8]		

Question			Answer	Marks	Guid
4	(i)	(a)			
			$v^2 = 250^2 + 210^2 - 2\times 250\times 210\cos 30°$	M1	Equation for v
			Magnitude is $125\,\text{m s}^{-1}$ (3 sf)	A1	
			$\frac{\sin\theta}{210} = \frac{\sin 30°}{125.2}$ $\theta = 57.0°$	M1	Equation for a relevant angle
			Bearing is 343° (3 sf)	A1 **[4]**	
		OR	${}_U\mathbf{v}_P = \begin{pmatrix} 250\sin 40° \\ 250\cos 40° \end{pmatrix} - \begin{pmatrix} 210\sin 70° \\ 210\cos 70° \end{pmatrix}$ $= \begin{pmatrix} -36.64 \\ 119.7 \end{pmatrix}$		M1 Subtracting components A1
			Magnitude is $125\,\text{m s}^{-1}$ Bearing is 343°		M1 Finding magnitude or bearing A1 Both correct

Question			Answer	Marks	Guid
4	(i)	(b)	As viewed from P  Shortest distance is $15000\sin 73°$ $=14300$ m (3 sf)	M1 A1 **[2]**	Or other complete method for distance
4	(ii)	(a)	 $\cos\phi = \frac{160}{250}$ $\phi = 50.2°$ Bearing is $350°$	B1 M1 A1 A1 **[4]**	Relative velocity perpendicular to $\mathbf{v}_Q$ Or $\psi = 39.8°$

Question			Answer	Marks	Guid	
4	(ii)	(b)	As viewed from Q  Shortest distance is $15000\sin 10.2°$ $= 2660$ m (3 sf)	M1 A1 **[2]**	Or other complete method for distance	
5	(i)		$I_{AD} = \frac{1}{3}(0.6)(0.75)^2 \quad (= 0.1125)$	B1		
			$I_{AB} = I_{CD} = 0.1125 + 0.6(0.75^2 + 0.75^2) \quad (= 0.7875)$ $I_{BC} = 0.1125 + (0.6)(1.5)^2 \quad (= 1.4625)$ $I = 0.1125 + 2\times 0.7875 + 1.4625 = 3.15$	M1 M1 A1	 AG	M0 for $\frac{4}{3}(0.$
		OR	$I = 4(0.1125 + 0.6\times 0.75^2) + (2.4)(0.75)^2$ $= 1.8 + (2.4)(0.75)^2$ $= 3.15$		M1 for $0.1125 + (0.6)(0.75)^2$ M1 for $I_G + (2.4)(0.75)^2$ A1 AG	
				[4]		
5	(ii)		$-2.4\times 9.8\times 0.75\sin\theta = 3.15\dfrac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$	B1 M1	For $2.4\times 9.8\times 0.75\sin\theta$ Equation of rotational motion	
		OR	$\frac{1}{2}I\omega^2 - mgh\cos\theta = K$ $I\omega\dot{\omega} + 2.4\times 9.8\times 0.75\sin\theta\dot{\theta} = 0$		M1 Differentiating energy equation A1	
			$\dfrac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -5.6\sin\theta$	A1 **[3]**	AG	

Question			Answer	Marks	Guid
5	(iii)		When θ is small, $\sin\theta \approx \theta$	B1	
			$\frac{d^2\theta}{dt^2} \approx -5.6\theta$, which is (approx.) SHM	B1	
			Period $(\frac{2\pi}{\sqrt{5.6}})$ is 2.66 s (3 sf)	B1 [3]	*Accept* $\pi\sqrt{\frac{5}{7}}$ *etc*
5	(iv)		WD by couple is 25×1.2 $(=30)$	B1	
			Change in PE is $2.4\times9.8(0.75-0.75\cos1.2)$ $(=11.25)$	B1	
			$\frac{1}{2}(3.15)\omega^2 = 30-11.25$	M1	Equation involving KE, WD and PE
			Angular speed is $3.45\ \text{rad s}^{-1}$ (3 sf)	A1 [4]	
6	(i)		GPE is $(-)mg(2a\cos\theta)\cos\theta$	B1	or $mg(a+a\cos2\theta)$
			EPE is $\frac{2mg}{2(\frac{1}{2}a)}(2a\cos\theta-\frac{1}{2}a)^2$	M1 A1	Using $\frac{\lambda x^2}{2l}$ (allow one error)
			$V = 2mga(4\cos^2\theta-2\cos\theta+\frac{1}{4})-2mga\cos^2\theta$ $= mga(6\cos^2\theta-4\cos\theta+\frac{1}{2})$	A1 [4]	AG
6	(ii)		$\frac{dV}{d\theta} = mga(-12\cos\theta\sin\theta+4\sin\theta)$	B1	or $mga(-6\sin2\theta+4\sin\theta)$
			Positions of equilibrium occur when $\frac{dV}{d\theta}=0$	M1	
			$\theta=0$ and $\theta=\cos^{-1}\frac{1}{3}$ $(=1.23$ or $70.5°)$ (Hence two positions)	A1A1 [4]	*Can be awarded when B1 has not been given*

Question			Answer	Marks	Guid
6	(iii)		$\frac{d^2V}{d\theta^2} = mga(-12\cos^2\theta + 12\sin^2\theta + 4\cos\theta)$	B1	FT if comparable
				M1	Considering the sign of $\frac{d^2V}{d\theta^2}$
			When $\theta = 0$, $\frac{d^2V}{d\theta^2} = -8mga < 0$		
			so this position is unstable	A1	CWO
			When $\theta = \cos^{-1}\frac{1}{3}$, $\frac{d^2V}{d\theta^2} = mga\left(-12\times\frac{1}{9} + 12\times\frac{8}{9} + 4\times\frac{1}{3}\right) = \frac{32}{3}mga > 0$		
			so this position is stable	A1 [4]	CWO
7	(i)		$I = \frac{1}{3}m\left[(3a)^2 + (4a)^2\right] + m(5a)^2$	B1	For $I_G = \frac{1}{3}m\left[(3a)^2 + (4a)^2\right]$
				M1	For $I_G + m(AG)^2$
		OR	$I = \frac{4}{3}m(3a)^2 + \frac{4}{3}m(4a)^2$		B1 For $I_{AD} = \frac{4}{3}m(3a)^2, I_{AB} = \frac{4}{3}m(4a)^2$ M1 For $I_{AD} + I_{AB}$
			$I = \frac{100}{3}ma^2$	A1 [3]	
7	(ii)		$\frac{1}{2}I\omega^2 = mg(4a + 5a\sin\theta)$	M1	Equation involving KE and PE
			$\frac{50}{3}ma^2\omega^2 = mga(4 + 5\sin\theta)$	A1	FT
			Angular speed $\omega = \sqrt{\frac{3g(4 + 5\sin\theta)}{50a}}$	A1 [3]	AG

Question			Answer	Marks	Guid
7	(iii)		$mg(5a\cos\theta) = I\alpha$	M1	Equation of rotational motion
			Angular acceleration is $\frac{3g\cos\theta}{20a}$	A1	*Accept* $\frac{15g\cos\theta}{100a}$ *etc*
				[2]	
7	(iv)			M1	For radial acceleration $r\omega^2$
			$R - mg\sin\theta = m(5a)\omega^2$	A1	
			$R - mg\sin\theta = \frac{3}{10}mg(4+5\sin\theta)$		
			$R = \frac{1}{10}mg(12+25\sin\theta)$	A1	
				M1	For transverse acceleration $r\alpha$
			$mg\cos\theta - S = m(5a)\alpha$	A1	
			$mg\cos\theta - S = \frac{3}{4}mg\cos\theta$		
			$S = \frac{1}{4}mg\cos\theta$	A1	
				[6]	

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